

$Z \rightarrow b\bar{b}$ and Chiral Currents in Higgsless Models

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In this note we compute the flavor-dependent chiral-logarithmic corrections to the decay $Z \rightarrow b\bar{b}$ in the three site Higgsless model. We compute these corrections diagrammatically in the “gaugeless” limit in which the electroweak couplings vanish. We also compute the chiral-logarithmic corrections to the decay $Z \rightarrow b\bar{b}$ using an RGE analysis in effective field theory, and show that the results agree. In the process of this computation, we compute the form of the chiral current in the gaugeless limit of the three-site model, and consider the generalization to the N -site case. We elucidate the Ward-Takahashi identities which underlie the gaugeless limit calculation in the three-site model, and describe how the result for the $Z \rightarrow b\bar{b}$ amplitude is obtained in unitary gauge in the full theory. We find that the phenomenological constraints on the three-site Higgsless model arising from measurements of $Z \rightarrow b\bar{b}$ are relatively mild, requiring only that the heavy Dirac fermion be heavier than 1 TeV or so, and are satisfied automatically in the range of parameters allowed by other precision electroweak data.

I. INTRODUCTION

Higgsless models [1] of electroweak symmetry breaking provide effective low-energy theories of a strongly-interacting symmetry breaking sector [2, 3] which, in the case of “delocalized” fermions [4–10], can be consistent with electroweak precision measurements [11, 12]. The three-site model [11] is the minimal low-energy realization of a Higgsless theory. It includes only the lightest triplet of the extra vector mesons typically present in such theories; these are the mesons (denoted here by W'^{\pm} and Z') that are analogous to the ρ mesons of QCD. The three-site model retains sufficient complexity, however, to incorporate interesting physics issues related to fermion masses and electroweak observables. In particular the chiral logarithmic corrections – the one-loop contributions which dominate in the limit $M_{W'} \ll \Lambda$ where $M_{W'}$ are the masses of the new vector mesons and Λ is the cutoff of the effective theory – to the flavor-universal electroweak parameters αS and αT [13–16] in the three-site model were computed in references [17–19].

In this note we compute the flavor-dependent chiral logarithmic corrections to the process $Z \rightarrow b\bar{b}$ in the three-site model. We perform the computation diagrammatically in the “gaugeless” limit [20–23], in which the electroweak couplings vanish: here the corrections to the Z -boson coupling are related to the couplings of the bottom-quark to the neutral Nambu-Goldstone boson present in this limit. As we illustrate, the computation of the amplitude is complicated by the mixing of the light fermions with the heavy Dirac fermions present in the three-site model; such issues do not arise in many other theories beyond the Standard Model, such as the MSSM or models featuring extended electroweak gauge groups but no new fermions. We also compute the chiral-logarithmic corrections to the decay $Z \rightarrow b\bar{b}$ in effective field theory and show that the results agree with the diagrammatic computation. We find that the phenomenological constraints on the three-site Higgsless model arising from measurements of $Z \rightarrow b\bar{b}$ are relatively mild, requiring only that the heavy Dirac fermion be heavier than 1 TeV or so, and are satisfied automatically in the range of parameters allowed by other precision electroweak data [12]. In the appendices we display the three-site model couplings necessary to compute the one-loop flavor-dependent corrections to the $Z \rightarrow b\bar{b}$ decay rate, we compute the chiral current in a general N -site Higgsless model, we provide a description of the Ward-Takahashi identities that underlie the gaugeless limit calculation in the three-site model, and we report the results of a unitary-gauge calculation of the process.

II. THE THREE-SITE MODEL

As discussed above we will compute the *nonuniversal* correction to $Z \rightarrow b\bar{b}$ by examining the chiral current and the couplings of the neutral Nambu-Goldstone boson eaten by the Z to b -quarks in the “gaugeless” limit [20–23] of the three-site model [11]. The gaugeless three-site $SU(2)_L \times SU(2)_H \times SU(2)_R$ model is illustrated in Fig. 1, where $SU(2)_H$ is a “hidden” gauge-symmetry [27–30] and $SU(2)_{L,R}$ are global symmetries. The nonlinear sigma-model and

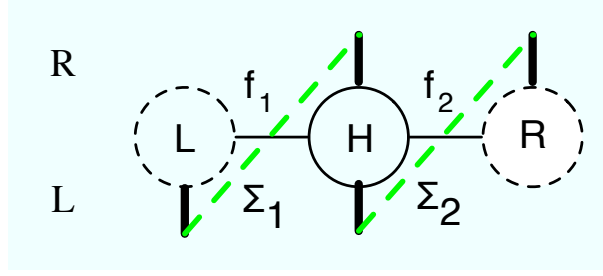


FIG. 1: The three site model in the gaugeless limit. The solid circle represents the strong $SU(2)_H$ gauge group with coupling \tilde{g} , and the dashed circles represent global $SU(2)_{L,R}$ groups. The left-handed fermions, denoted by the lower vertical lines and labeled $\psi_L^{(0),(1)}$ in the text, are located at the first two sites, and the right-handed fermions, denoted by the upper vertical lines and labeled $\psi_R^{(1)}$ and $(t_R^{(2)}, b_R^{(2)})$ in the text, are located at the last two sites. The dashed lines correspond to Yukawa couplings. We will denote the light mass-eigenstate fermion fields by (t, b) and the heavy ones by (T, B) .

gauge-theory kinetic-energy terms in this model are given by

$$\mathcal{L} = \sum_{i=1,2} \frac{f_i^2}{4} \text{tr} \left(D^\mu \Sigma_i^\dagger D_\mu \Sigma_i \right) - \frac{1}{4\tilde{g}^2} (\vec{W}'^{\mu\nu})^2, \quad (1)$$

where Σ_1 and Σ_2 are sigma-model fields parameterized by

$$\Sigma_{1,2} = \exp \left(\frac{2i\pi_{1,2}}{f_{1,2}} \right), \quad (2)$$

where $\pi_{1,2} \equiv \pi_{1,2}^a \sigma^a / 2$, and where $\vec{W}'^{\mu\nu}$ is the field-strength tensor of the $SU(2)_H$ gauge-group with gauge-fields W'_μ . The one-loop contributions described in section IV will be performed in 't Hooft-Feynman gauge for the hidden $SU(2)_H$ gauge-symmetry, and the appropriate gauge-fixing terms (and ghost terms, though these are not needed for the current computation) are also introduced though they are not displayed here.

The sigma-model fields transform as

$$\Sigma_1 \rightarrow L \Sigma_1 H^\dagger, \quad (3)$$

$$\Sigma_2 \rightarrow H \Sigma_2 R^\dagger, \quad (4)$$

under $SU(2)_L \times SU(2)_H \times SU(2)_R$, and hence the covariant derivatives above are given by

$$D^\mu \Sigma_1 = \partial^\mu \Sigma_1 + iW'^{a\mu} \Sigma_1 \frac{\sigma^a}{2}, \quad (5)$$

$$D^\mu \Sigma_2 = \partial^\mu \Sigma_2 - iW'^{a\mu} \Sigma_2 \frac{\sigma^a}{2}. \quad (6)$$

Here $f_{1,2}$ are the f -constants, the analogs of F_π in QCD, associated with the two $SU(2) \times SU(2)/SU(2)$ nonlinear sigma-models, and they satisfy the relation

$$\sqrt{2}G_F = \frac{1}{v^2} = \frac{1}{f_1^2} + \frac{1}{f_2^2} \approx \frac{1}{(250 \text{ GeV})^2}. \quad (7)$$

In the gaugeless limit, the $SU(2)_H$ vector bosons have mass

$$M_{W'}^2 = \frac{\tilde{g}^2(f_1^2 + f_2^2)}{4}. \quad (8)$$

As described in [11], we get a phenomenologically-acceptable low-energy electroweak model if we identify $SU(2)_L$ with the weak gauge-group and $U(1)_Y$ with the subgroup of $SU(2)_R$ associated with T_3 , and if we work in the limit where couplings satisfy $g_{W,Y} \ll \tilde{g}$, *i.e.* in the limit $M_{W,Z}^2 \ll M_{W'}^2$.

The three-site model also incorporates the ordinary quarks and leptons, and requires the additional heavy vectorial $SU(2)_H$ fermions that mirror the light fermions. These heavy Dirac fermions are the analogs of the lowest Kaluza-Klein (KK) fermion modes which would be present in an extra-dimensional theory. The Yukawa couplings for the third-generation are written

$$\mathcal{L}_f = -m_1 \bar{\psi}_L^{(0)} \Sigma_1 \psi_R^{(1)} - M \bar{\psi}_R^{(1)} \psi_L^{(1)} - \bar{\psi}_L^{(1)} \Sigma_2 \begin{pmatrix} m'_t & \\ & 0 \end{pmatrix} \begin{pmatrix} t_R^{(2)} \\ b_R^{(2)} \end{pmatrix} + h.c. , \quad (9)$$

where we use the notation of [12] and we treat the bottom-quark as massless.¹ The transformation properties of the fermions under $SU(2)_L \times SU(2)_H \times SU(2)_R$ are given by

$$\psi_L^{(0)} \rightarrow L \psi_L^{(0)} , \quad (10)$$

$$\psi_{R,L}^{(1)} \rightarrow H \psi_{R,L}^{(1)} , \quad (11)$$

$$\begin{pmatrix} t_R^{(2)} \\ b_R^{(2)} \end{pmatrix} \rightarrow R \begin{pmatrix} t_R^{(2)} \\ b_R^{(2)} \end{pmatrix} . \quad (12)$$

We will work in the limit where $M \gg m_1, m'_t$, in which the heavy fermions are approximately the $\psi_{L1,R1}$ doublets with a mass approximately equal to M . The couplings between the fermions and the Nambu-Goldstone bosons that are necessary for the one-loop computations are summarized in Appendix A.

The ratio $\epsilon_L \equiv m_1/M$ controls the “delocalization” of the left-handed fermions, *i.e.* the amount to which the light left-handed mass eigenstate fields are admixtures of fermions at the first two sites; this parameter can be adjusted to eliminate the potentially dangerous tree-level contributions to αS [4–10]. Therefore, at tree-level in the three-site model, ϵ_L is taken to be flavor-universal and all of the flavor-breaking is encoded in the values of Yukawa couplings to the right-handed fermions, which transform under $SU(2)_R$. The three-site model at tree-level has precisely the same flavor structure as the standard model: all of the tree-level couplings of the left-handed fermions to the gauge bosons are flavor-diagonal and equal, and flavor-changing neutral currents are suppressed [11]. As we will see, however, in the case of the third generation the Yukawa couplings proportional to m'_t will distinguish the top- and bottom-quarks from the light generations, leading to flavor-dependence at one loop. Before turning to this issue, however, we consider how the W - and Z -bosons – which, in the gaugeless limit, are treated as external fields – couple to the fermions and Nambu-Goldstone-bosons.

III. CHIRAL CURRENTS IN THE “GAUGELESS” LIMIT

In the gaugeless limit, one treats the Z -boson as an external field coupled to the current

$$j_Z^\mu = j_{3L}^\mu - j_Q^\mu \sin^2 \theta_W , \quad (13)$$

with strength

$$g_Z = \frac{e}{\sin \theta_W \cos \theta_W} . \quad (14)$$

A crucial question² is precisely what is meant by the current j_{3L}^μ . We begin by computing this current at tree-level, which is most easily done in “unitary” gauge for the group $SU(2)_H$, in which the link fields satisfy the property that $f_1 \pi_1 = f_2 \pi_2$. In this gauge, it is easy to show [31] that

$$\Sigma = \Sigma_1 \cdot \Sigma_2 = \exp \left(\frac{2i\pi}{v} \right) , \quad (15)$$

where

$$\frac{\pi}{v} = \frac{\pi_1}{f_1} + \frac{\pi_2}{f_2} + \dots , \quad (16)$$

¹ Similar terms must be introduced for all of the light quarks and leptons [12] as well, but these terms will not play a role in what follows.

² Note that the same question does not arise for j_Q^μ which is unbroken and therefore unrenormalized. We shall explicitly see how the difference between j_L^μ and j_Q^μ arises below.

is the non-linear sigma-model field depending only on the massless Nambu-Goldstone bosons (π_{W^\pm} and π_Z), which remain in the limit $g_{W,Y} \rightarrow 0$. From eqns. (3) and (4) we expect contributions to j_L^μ from the Σ , which are found to have the usual nonlinear sigma-model form [31]

$$j_{L,GB}^{\mu a} = -i \frac{v^2}{2} \text{tr}(T^a \Sigma \partial^\mu \Sigma^\dagger) , \quad (17)$$

and, from eqn. (10), we expect contributions of conventional form from the ψ_{L0} fermions. In addition, there are contributions to j_{3L}^μ proportional to the heavy W' gauge-bosons [31]. At low-momentum, the contributions proportional to the W' boson give rise to contributions to the chiral current from the fermions $\psi_{L1,R1}$. Rather than pursue this calculation, we will compute the entire fermionic contribution to the chiral current more directly.

Consider the transformation of the sigma-model fields under a global $SU(2)_L \times SU(2)_R$ transformation. In unitary gauge, from eqns. (3) and (4), the transformation laws of the fields are given by

$$\Sigma_1 \rightarrow L \Sigma_1 H(L, R, \pi)^\dagger , \quad (18)$$

$$\Sigma_2 \rightarrow H(L, R, \pi) \Sigma_2 R^\dagger , \quad (19)$$

where $H(L, R, \pi)$ represents the $SU(2)_H$ gauge-transformation which, following a global L and R transformation, is necessary to return to unitary gauge. As noted, $H(L, R, \pi)$ depends on L , R and the pion fields in $\Sigma_{1,2}$ as well, in such a way that the relation $f_1 \pi_1 = f_2 \pi_2$ continues to be satisfied. From eqn. (11) we see that the fields $\psi_{R1,L1}$ transform through $H(L, R, \pi)$, and therefore transform nonlinearly under $SU(2)_L \times SU(2)_R$. In fact, the transformation properties of $\psi_{R1,L1}$ are precisely those of matter fields in the Callan-Coleman-Wess-Zumino (CCWZ) construction³ [32, 33]. Note, however, that the ψ_{L0} and ψ_{R2} fields transform linearly under $SU(2)_L \times SU(2)_R$ – and hence, in the computation of the current, these fields will contribute in the conventional manner.

Under an infinitesimal left-handed transformation

$$\delta \psi_L^{(0)} = i \alpha_L \psi_L^{(0)} , \quad (20)$$

$$\delta \psi_{R,L}^{(1)} = i \eta \psi_{R,L}^{(1)} , \quad (21)$$

where $\alpha_L = \alpha_L^a \sigma^a / 2$ and $\eta = \eta^a \sigma^a / 2$ denote 2×2 hermitian traceless matrices, and where α_L^a and η_L^a are small parameters. From eqns. (18) and (19) we see that, to lowest order in α , η , and $\pi_{1,2}$

$$\frac{2\pi_1}{f_1} \rightarrow \frac{2\pi_1}{f_1} + \alpha_L - \eta , \quad (22)$$

$$\frac{2\pi_2}{f_2} \rightarrow \frac{2\pi_2}{f_2} + \eta . \quad (23)$$

Imposing the relation $f_1 \pi_1 = f_2 \pi_2$, we find

$$\eta = \frac{f_1^2}{f_1^2 + f_2^2} \alpha_L + \mathcal{O}(\alpha_L^2, \alpha_L \pi) , \quad (24)$$

and hence, from eqns. (20) and (21), the fermionic contributions to the left-handed current are

$$j_L^{a\mu} = \bar{\psi}_L^{(0)} \frac{\sigma^a}{2} \gamma^\mu \psi_L^{(0)} + \frac{f_1^2}{f_1^2 + f_2^2} \left(\bar{\psi}_L^{(1)} \frac{\sigma^a}{2} \gamma^\mu \psi_L^{(1)} + \bar{\psi}_R^{(1)} \frac{\sigma^a}{2} \gamma^\mu \psi_R^{(1)} \right) . \quad (25)$$

Note that for $f_1 = f_2 = \sqrt{2}v$, the fermions $\psi_{L,R}^{(1)}$ couple to the current with half the strength of $\psi_L^{(0)}$ – this explains the size of weak-boson couplings of the vector (KK) fermions in the three-site model; see eqn. (5.10) of [11].⁴ A generalization of this result to the N -site global moose model appears in Appendix B.

³ For a recent review connecting deconstruction and the CCWZ procedure, see [34].

⁴ We could have obtained precisely the same results using the procedure of [31]: in this case, the chiral current would contain contributions only from the pions, the fermions ψ_{L0} , and the W' -bosons. Integrating out the W' bosons, we recover the $\psi_{L1,R1}$ contributions we find in eq. (25).

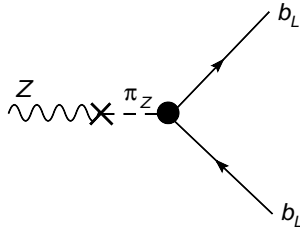


FIG. 2: The Nambu-Goldstone boson contributions to the current j_{3L}^μ give rise to corrections to the $Zb\bar{b}$ vertex [21, 22]. In this figure, the black circle represents vertices in the effective action of the form $\partial^\mu \pi_Z \bar{b}_L \gamma_\mu b_L$.

The difference in the couplings of the site-0 and site-1 fermions to $j_L^{a\mu}$ is precisely the reason why delocalization [9] can shift the size of the fermion couplings to the gauge-bosons and allow – in the case of ideal delocalization – for $\alpha S = 0$. At tree-level, the left-handed light fermion eigenstate is given (up to corrections of order ϵ_L^3) by [11]

$$\psi_L = - \left(1 - \frac{\epsilon_L^2}{2} \right) \psi_L^{(0)} + \epsilon_L \psi_L^{(1)} , \quad (26)$$

and, hence, in the gaugeless limit the light-fermion contribution to the left-handed current is given by⁵

$$j_L^{a\mu} \supset \left(1 - \frac{\epsilon_L^2 f_2^2}{f_1^2 + f_2^2} \right) \bar{\psi}_L \gamma^\mu \frac{\sigma^a}{2} \psi_L . \quad (27)$$

In addition, the rotation in eqn. (26) also yields couplings of the left-handed current to mixtures of the light- and heavy-fermion eigenstates. As we will see below, these “off-diagonal” Z -boson couplings will be important in the computation of the one-loop correction to the $Z \rightarrow b\bar{b}$ decay rate. Note also that there is no change in the weak-charge of the light fermions in the limit that $f_1 \rightarrow \infty$, since in this case $SU(2)_L \times SU(2)_H \rightarrow SU(2)_{L+H}$ and the ψ_{L1} couples (see eqn. (25)) to the chiral current in the same⁶ way as ψ_{L0} .

By contrast, in the case of the unbroken electromagnetic current,

$$j_Q^\mu = \bar{\psi}_L^{(0)} Q \gamma^\mu \psi_L^{(0)} + \bar{\psi}_L^{(1)} Q \gamma^\mu \psi_L^{(1)} + \bar{\psi}_R^{(1)} Q \gamma^\mu \psi_R^{(1)} + \bar{\psi}_R^{(2)} Q \gamma^\mu \psi_R^{(2)} , \quad (28)$$

where $Q = \text{diag}(2/3, -1/3)$ is the quark charge-matrix. In this case all fermions couple to the photon in the same way, and fermion delocalization (mixing) cannot change the electric charge of the fermions.

IV. ONE-LOOP CORRECTIONS

As previously noted, in the gaugeless limit, one computes the couplings of external Z bosons to the current $j_{3L}^\mu - j_Q^\mu \sin^2 \theta_W$. The current j_Q^μ is conserved and therefore unrenormalized and flavor-universal. The flavor non-universal couplings to the Z boson occur because j_{3L}^μ is a current that corresponds to a spontaneously broken symmetry, and therefore arise from the non-linear sigma model currents in eqn. (17). As we will show below, there are flavor-dependent contributions in the effective action to operators of the form

$$A \partial^\mu \pi_Z \bar{b}_L \gamma_\mu b_L . \quad (29)$$

Through the diagram illustrated in Fig. 2, such an operator shifts⁷ the left-handed $Zb\bar{b}$ coupling to

$$g_Z \left(-\frac{1}{2} + \delta g_L^{b\bar{b}} + \frac{1}{3} \sin^2 \theta_W \right) , \quad (30)$$

⁵ Eqn. (27) is sufficient only to compute the leading order couplings of $j_L^{a\mu}$ to the Z -boson. Once the weak gauge-interactions are turned on, $g_{W,Y} \neq 0$, there are additional corrections (of order g_W^2/\bar{g}_1^2) arising from the admixture of the site-1 gauge-boson in the light gauge-boson mass eigenstate. While these corrections are not included in the gaugeless limit, they are flavor universal (to leading non-trivial order) and do not affect the ratio $\Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})$.

⁶ This result is analogous to the GIM cancellations that occur in the mixing of left-handed quarks in the standard model – since all the quarks have the same left-handed charges, such mixing does not result in flavor-changing neutral couplings nor does it change the tree-level Z -coupling of any of the quarks.

⁷ This is, essentially, a diagrammatic interpretation of the Ward-Takahashi Identity argument given in [21, 22]; see Appendix C.

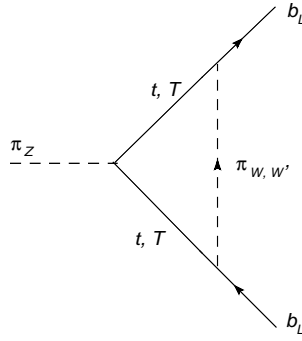


FIG. 3: The flavor-dependent vertex corrections to the $Z \rightarrow b\bar{b}$ branching ratio, as computed in the gaugeless limit of the three-site model. Here π_Z and π_W denote the neutral and charged Nambu-Goldstone bosons which remain in the gaugeless limit. We perform this calculation in 't Hooft-Feynman gauge for $SU(2)_H$, and the $\pi_{W'}$ denote the unphysical Nambu-Goldstone bosons eaten by the heavy W' bosons; the flavor-dependent contributions from the W' gauge-bosons are suppressed by an additional factor of $M_{W'}^2/M^2$. In this diagram we denote the heavy Dirac partners of the top-quark by T . The contribution with intermediate t quarks and exchange of the π_W boson yields the usual standard model one-loop correction [21, 22]. The other contributions are new in the three-site model.

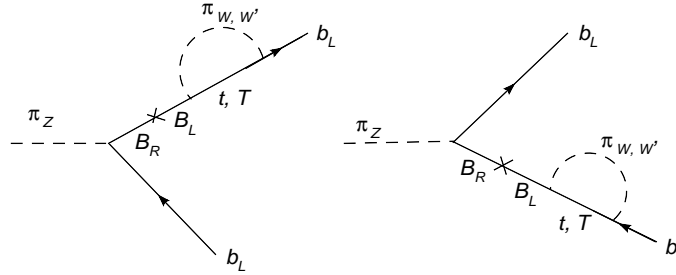


FIG. 4: Wavefunction mixing contributions to $Z \rightarrow b\bar{b}$ in the three site model. The B fermions are the heavy Dirac partners of the bottom-quark.

where g_Z and $\sin^2 \theta_W$ are flavor-independent, and where the flavor-dependent correction is

$$\delta g_L^{b\bar{b}} = \frac{v}{2} A . \quad (31)$$

The vertex diagrams (in 't Hooft-Feynman gauge) leading to flavor-dependent contributions⁸ to the operator in eqn. (29) are illustrated in Fig. 3. The triangle contribution with π_W -exchange and two intermediate top-quarks yields the usual gaugeless standard model correction⁹ [21–23]

$$(\delta g_L^{b\bar{b}})_{sm} = \frac{m_t^2}{16\pi^2 v^2} . \quad (32)$$

On the other hand, the triangle diagrams including contributions from the heavy Dirac partners of the top-quarks and/or the exchange of the $\pi_{W'}$ boson, yield the correction

$$(\delta g_L^{b\bar{b}})_{3-site\ vertex} = -\frac{1}{2(4\pi)^2} \frac{m_t^2}{v^2} \frac{f_1^2 f_2^2}{(f_1^2 + f_2^2)^2} \log \left(\frac{M^2}{m_t^2} \right) , \quad (33)$$

⁸ We neglect here, for example, vertex diagrams involving the exchange of $\pi_{Z, Z'}$ and intermediate b or B quarks: in the limit in which we ignore the b -quark mass, these contributions are the same for all flavors of quarks. Also, in 't Hooft-Feynman gauge the flavor-dependent contributions arising from W' exchange are suppressed since the heavy Dirac partners of all of the fermions are nearly degenerate. As we discuss at the end of this section, there are subleading – suppressed by $M_{W'}^2/M^2$ – contributions from the additional diagrams in figure 5.

⁹ There are additional one-loop corrections [24–26] proportional to weak couplings (and at most logarithmically dependent on m_t) which cannot be computed in the gaugeless limit.

where we have used eqn. (7) and the relation

$$m_t \approx \frac{m_1 m'_t}{M} . \quad (34)$$

In addition to the vertex corrections, there are flavor-dependent wavefunction mixing contributions¹⁰ which must be added, as illustrated in Fig. 4. These contributions exist in the three-site model because of the existence of the $\pi_Z \bar{b}_L B_R$ vertex, which couples the b to its heavy Dirac partner B (see eqn. (A13)). There is no analogous contribution in the standard model, because the $\pi_Z \bar{b}_L b_R$ vertex vanishes in the limit of zero bottom-quark mass. In Appendix C, we show how these terms arise from the Ward-Takahashi identity of the three-site model. Here we present an alternative derivation to show directly how these extra contributions give rise to operators of the form in eqn. (29) after integrating out the heavy B field. Including the wavefunction mixing, we may write the effective action as

$$\mathcal{L} = \bar{b}_L i \not{\partial} b_L + \bar{B} i \not{\partial} B - M \bar{B} B + \eta \bar{B}_L i \not{\partial} b_L + i g_{\pi b B} \pi_Z \bar{b}_L B_R + h.c. + \dots , \quad (35)$$

where η is a small-parameter of one-loop order. The linear terms in the equations of motion for the B field are then

$$i \not{\partial} B_L - M B_R + i \eta \not{\partial} b_L + \dots = 0 , \quad (36)$$

$$i \not{\partial} B_R - M B_L + \dots = 0 , \quad (37)$$

where the ellipses refer to (interaction) terms with more than one field. Integrating out the B field in the large- M limit, we find¹¹

$$B_R = i \frac{\eta}{M} \not{\partial} b_L + \dots . \quad (38)$$

Plugging this expression into the $\pi_Z \bar{b}_L B_R$ (and Hermitian conjugate) coupling yields the operator in eqn. (29). The wavefunction mixing diagrams are logarithmically divergent. Performing the calculation using dimensional regularization and using $\overline{\text{MS}}$ we find that the wavefunction mixing contributions then yield

$$(\delta g_L^{b\bar{b}})_{3\text{-site wavefunction}} = + \frac{1}{2(4\pi)^2} \frac{m_t^2}{v^2} \frac{f_1^2 f_2^2}{(f_1^2 + f_2^2)^2} \left(\log \left(\frac{\mu^2}{m_t^2} \right) + \frac{3}{2} \right) , \quad (39)$$

where μ is the regularization mass.

Adding all of the flavor-dependent contributions from eqns. (32), (33) and (39), we find the total contribution

$$\delta g_L^{b\bar{b}} = \frac{m_t^2}{(4\pi)^2 v^2} \left[1 + \frac{f_1^2 f_2^2}{2(f_1^2 + f_2^2)^2} \left(\log \left(\frac{\mu^2}{M^2} \right) + \frac{3}{2} \right) + \delta g_L^{b\bar{b}}(\mu) \right] , \quad (40)$$

where $\delta g_L^{b\bar{b}}(\mu)$ represents the three-site model counterterm, renormalized at scale μ , required to renormalize the theory appropriately. As shown in [12], and discussed further in the next section, at one-loop there are flavor-dependent renormalizations of the heavy Dirac masses M in eqn. (9), and $\delta g_L^{b\bar{b}}(\mu)$ represents the effect of the counterterm necessary to implement this renormalization.

Two properties of this result are worth commenting on. First, note that the additional three-site contributions vanish in the limit that f_1 or $f_2 \rightarrow \infty$ with v held fixed. This is reasonable since the three-site model reduces to the electroweak chiral Lagrangian [35–39] in this limit, and the $Z \rightarrow b\bar{b}$ corrections must therefore reduce to those of the standard model. Second, note that the corrections proportional to $\log(m_t)$ cancel when we add the three-site vertex and wavefunction mixing contributions – this confirms the effective field-theory argument given in [11], which noted that once the heavy fermions were integrated out there were no operators in the effective theory whose scaling could affect the size of the $Z b\bar{b}$ coupling.

The results described above are the leading-order flavor-dependent contributions, in 't Hooft-Feynman gauge for the hidden $SU(2)_H$ gauge theory, arising from the diagrams in figures 3 and 4; there are also subleading diagrams as illustrated in fig. 5. We have checked our results by computing the one-loop corrections to $Z \rightarrow b\bar{b}$ in the full three-site model in unitary gauge, as described in Appendix D.

¹⁰ Again, as in the case of the vertex diagrams, the flavor-dependent contributions from the diagrams with W' exchange in figure 5 are suppressed.

¹¹ An alternative procedure would be to diagonalize the kinetic energy and mass terms in (35). In this case the analysis is a bit more complicated, though the S -matrix that arises is equivalent. Note that on-shell matrix elements of the operator in eqn. (29) are of order m_b . One must include a non-zero mass for the bottom-quark and carefully keep track of terms of order m_b ; doing so, one finds that B_R mixes with b_R , and yields an effect equivalent to the one we compute above. Note that although one must keep terms proportional to m_b in this procedure, the final correction to the $Z \rightarrow b\bar{b}$ amplitude is not proportional to m_b .

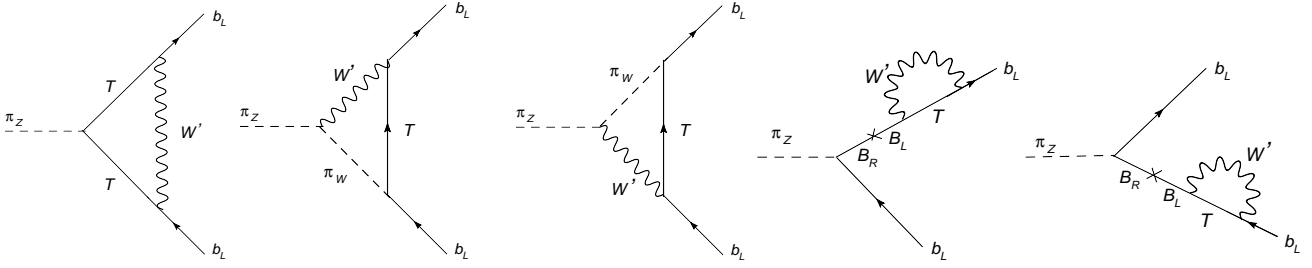


FIG. 5: These W' -exchange diagrams yield flavor non-universal contributions which are suppressed, relative to the contributions we have kept, by $M_{W'}^2/M^2$.

V. RGE ANALYSIS

It is interesting to see how the logarithmic term in eqn. (40) can be reproduced using the RGE analysis of ref. [12]. In this case, one analyzes the three-site model in the limit $\Lambda \gg M \gg M_{W'}$ in effective field theory. As described in [12], we define the parameters in eqns. (1) and (9) in terms of their values at the cutoff scale Λ . For the energy regime between Λ and M , one considers the one-loop running of the operators in the full three-site model. At scale M , one “integrates out” the heavy fermions and constructs an effective theory with one-site delocalization [9]. Subsequently, one computes the running of the operators from the scale M to the scale $M_{W'}$. At this stage, one integrates out the heavy W' fields and matches the theory to the electroweak chiral lagrangian [35–39]. Lastly, one runs in the electroweak chiral lagrangian to energies of order M_Z to analyze the electroweak processes of interest.

The only flavor-dependent terms arise from the Yukawa couplings proportional to m'_t . As shown in ref. [12], these terms give rise at one-loop to a flavor-dependent renormalization of the masses of the heavy Dirac fermions in the running between the cutoff scale Λ and the heavy fermion scale M . This flavor-dependent renormalization of the $\psi_L^{(1)}$ wavefunction arises from the π_2 interactions eqn. (9). Conventionally normalizing the third-generation $\psi_L^{(1)}$ fields then leads to a flavor-dependent shift in the Dirac mass M , and the RGE running of this parameter.

The quantity of phenomenological interest is the ratio $\epsilon_L = m_1/M$, which defines the delocalization of the light left-handed fermions. We find [12] the running

$$\mu \frac{d}{d\mu} \left(\frac{m_1}{M} \right)_{3rd} = \frac{1}{(4\pi)^2} \frac{m_1}{M} \left(\frac{9}{2} \tilde{g}^2 - \frac{9}{2} \frac{m_1^2}{f_1^2} - \frac{m_t'^2}{f_2^2} \right), \quad (41)$$

for the third generation of quarks, while

$$\mu \frac{d}{d\mu} \left(\frac{m_1}{M} \right)_{1st} = \frac{1}{(4\pi)^2} \frac{m_1}{M} \left(\frac{9}{2} \tilde{g}^2 - \frac{9}{2} \frac{m_1^2}{f_1^2} \right) \quad (42)$$

for the first and second generations of quarks. Assuming universality at the cutoff scale Λ

$$\left(\frac{m_1}{M} \right)_{1st} \Big|_{\mu=\Lambda} = \left(\frac{m_1}{M} \right)_{3rd} \Big|_{\mu=\Lambda}, \quad (43)$$

we find that the renormalization group equations Eqs.(41)-(42) induce a flavor-non-universal correction

$$\Delta \epsilon_L^2 \equiv \left(\frac{m_1}{M} \right)_{3rd}^2 \Big|_{\mu=M} - \left(\frac{m_1}{M} \right)_{1st}^2 \Big|_{\mu=M} = \frac{1}{(4\pi)^2} \frac{m_t'^2}{f_2^2} \left(\frac{m_1}{M} \right)^2 \ln \frac{\Lambda^2}{M^2}, \quad (44)$$

at the KK fermion mass scale M . Below the scale M all subsequent evolution is flavor-universal.

The effect of this flavor-dependent renormalization is to shift the value of ϵ_L for the third-generation quarks

$$\epsilon_L^2 \rightarrow \epsilon_L^2 + \frac{1}{(4\pi)^2} \frac{m_t'^2}{f_2^2} \log \frac{\Lambda^2}{M^2}, \quad (45)$$

where we have simplified by using eqn. (34). Using eqns. (7), (27), and (30), we then find the correction

$$(\delta g_L^{b\bar{b}})_{3-site} = + \frac{1}{2(4\pi)^2} \frac{m_t'^2}{v^2} \frac{f_1^2 f_2^2}{(f_1^2 + f_2^2)^2} \log \left(\frac{\Lambda^2}{M^2} \right). \quad (46)$$

The result of eqn. (46) is in agreement with the second term in eqn. (40) with the identifications

$$\log \Lambda^2 = \log \mu^2 + \frac{3}{2} , \quad (47)$$

$$\delta g_L^{b\bar{b}}(\Lambda) = 0 . \quad (48)$$

From the discussion above, we see that the $\delta g_L^{b\bar{b}}(\mu)$ represents the dependence of the $Z \rightarrow b\bar{b}$ amplitude on the (flavor-dependent) heavy Dirac mass, and the condition $\delta g_L^{b\bar{b}}(\Lambda) = 0$ expresses the choice (eqn. (43)) of flavor-universal Dirac masses when the theory is renormalized at the cutoff scale Λ .

VI. LIMITS FROM R_b

The best way to compare our expressions (eqn. (46)) for the shifted $Zb\bar{b}$ coupling to experiment is by calculating the ratio R_b :

$$R_b \equiv \frac{\Gamma(Z \rightarrow b\bar{b})}{\Gamma(Z \rightarrow \text{hadrons})} . \quad (49)$$

This ratio can be evaluated as

$$R_b \simeq \frac{g_{bL}^2 + g_{bR}^2}{\sum_{f=u,d,c,s,b} (g_{fL}^2 + g_{fR}^2)} , \quad (50)$$

where g_{fL} (g_{fR}) denotes the Z boson coupling to the left (right) handed f quark. Following Ref. [23], we may decompose the $Zb_L\bar{b}_L$ couplings (as elsewhere in this paper) into standard model and New Physics pieces,

$$g_{bL} = g_{bL}^{\text{SM}} + \delta g_L^{\text{NP}} . \quad (51)$$

We may express the New Physics effect on R_b in terms of δg_L^{NP} [23]

$$\delta R_b = 2R_b(1 - R_b) \frac{g_{bL}}{g_{bL}^2 + g_{bR}^2} \delta g_L^{\text{NP}} . \quad (52)$$

To leading order we may simplify this expression by inserting the value of R_b predicted by the standard model [40]

$$R_b^{\text{SM}} = 0.21584 \pm 0.00006 . \quad (53)$$

as well as the standard model values¹² of g_{bL} and g_{bR}

$$g_{bL} = -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W , \quad g_{bR} = \frac{1}{3} \sin^2 \theta_W , \quad \sin^2 \theta_W \simeq 0.23 , \quad (54)$$

to obtain

$$\delta R_b \simeq -0.774 \times \delta g_L^{\text{NP}} . \quad (55)$$

In order to place a limit on the size of the coupling shift due to New Physics, we compare the result above with the data. The observed value of R_b [40] is

$$R_b^{\text{obs}} = 0.21629 \pm 0.00066 , \quad (56)$$

and the gap between this and the standard model value (56) is

$$\delta R_b^{\text{obs}} \equiv R_b^{\text{obs}} - R_b^{\text{SM}} = (4.5 \pm 6.6) \times 10^{-4} . \quad (57)$$

Comparing this with eqn. (55) yields the general constraint

$$\delta g_L^{\text{NP}} = (-5.8 \pm 8.6) \times 10^{-4} , \quad (58)$$

¹² The level of accuracy for $\sin^2 \theta_W$ in eqn. (54) is sufficient to compute the size of the one-loop correction to R_b to the required accuracy.

on the shift any New Physics may induce in the $Zb\bar{b}$ coupling.

For the three-site model in particular, δg_L^{NP} corresponds to the coupling shift we obtained in eqn. (46):

$$\delta g_L^{\text{NP}} = \frac{m_t^2}{(4\pi)^2 v^2} F = \frac{\sqrt{2} G_F m_t^2}{(4\pi)^2} F, \quad (59)$$

with

$$F = \frac{f_1^2 f_2^2}{2(f_1^2 + f_2^2)^2} \ln \frac{\Lambda^2}{M^2}. \quad (60)$$

If we insert the values

$$m_t \simeq 171 \text{ GeV}, \quad G_F \simeq 1.166 \times 10^{-5} \text{ GeV}^{-2}, \quad (61)$$

we obtain the following bound on F

$$F = -0.19 \pm 0.28. \quad (62)$$

Since F is theoretically constrained to be positive, we need to be a bit careful when we deduce a 95% CL bound on F . Following [23], we apply the method proposed in [41], including the information in its Table X; our result is

$$F < 0.38 \quad (95\% \text{ CL}). \quad (63)$$

From this, we may derive a bound on Λ/M . For example, in the case $f_1 = f_2 = \sqrt{2}v$ (which corresponds to maximal unitarity delay [11]), we find

$$\frac{\Lambda}{M} < 4.6 \quad (95\% \text{ CL}). \quad (64)$$

We expect a Λ of order 4 TeV or less, since $\Lambda \leq 4\pi f_{1,2} \approx 4\sqrt{2}\pi v$, and therefore we obtain the constraint that the heavy fermion masses should be at least of order one TeV – a constraint satisfied automatically by the three-site model in the range of parameter space allowed by other precision electroweak data [11, 12, 42].

VII. CONCLUSIONS

We have computed the flavor-dependent chiral-logarithmic corrections to the decay $Z \rightarrow b\bar{b}$ in the three site Higgsless model and have demonstrated that the diagrammatic calculation in the gaugeless limit agrees with an RGE analysis of the effective theory. We have shown the necessity of carefully incorporating the effects of mixing between the light- and heavy-fermions in the computation of this result; such effects are not present in many other theories beyond the Standard Model, such as the MSSM or models featuring extended electroweak gauge groups but no new fermions. Comparing our three-site model result

$$\delta g_L^{b\bar{b}} = \frac{m_t^2}{(4\pi)^2 v^2} \left[1 + \frac{f_1^2 f_2^2}{2(f_1^2 + f_2^2)^2} \log \left(\frac{\Lambda^2}{M^2} \right) \right],$$

with the data on R_b yields the rather mild constraint that the heavy fermions have masses of at least 1 TeV. This limit is automatically satisfied by the three-site model in the range of parameter space allowed by other precision electroweak data [11, 12, 42]. Moreover, the form we obtain for the chiral currents in an N -site global moose model with fermion delocalization suggests that the effects on R_b in such models (and therefore in continuum models as well) will be similar.

It is interesting to note the contrast between our results and those for Higgsless models without delocalization (those in which the W' is not fermiophobic). We found that in the three-site model, corrections proportional to $\ln m_t$ cancel between the vertex and wavefunction mixing contributions. Since the effective theory is valid only to $\Lambda \approx 4\pi\sqrt{2}v \approx 4 \text{ TeV}$, while precision electroweak data force M to lie above 1.8 TeV or so [11, 12, 42], the remaining chiral log factor $\ln(\Lambda^2/M^2)$ cannot be large. However, in generic extra dimensional models when one integrates out the KK modes [23], corrections proportional to $\ln(M_{W'}^2/m_t^2)$ can persist and lead to more stringent experimental constraints.

Acknowledgments

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APPENDIX A: THREE-SITE MODEL COUPLINGS, MASSES, AND EIGENSTATES

In this appendix we present the couplings, masses, and mass eigenstate fields required in the computation of $Z \rightarrow b\bar{b}$ in the gaugeless limit. The quark mass matrices of this model are

$$(\bar{b}_L^{(0)}, \bar{b}_L^{(1)}) \begin{pmatrix} m_1 & 0 \\ M & 0 \end{pmatrix} \begin{pmatrix} b_R^{(1)} \\ b_R^{(2)} \end{pmatrix}, \quad (\bar{t}_L^{(0)}, \bar{t}_L^{(1)}) \begin{pmatrix} m_1 & 0 \\ M & m'_t \end{pmatrix} \begin{pmatrix} t_R^{(1)} \\ t_R^{(2)} \end{pmatrix}. \quad (\text{A1})$$

Denoting the heavy and light mass eigenstates by (t, b) and (T, B) , we diagonalize the matrices by

$$\begin{pmatrix} b_L \\ B_L \end{pmatrix} = V_{bL} \begin{pmatrix} b_L^{(0)} \\ b_L^{(1)} \end{pmatrix}, \quad V_{bL} \simeq \begin{pmatrix} -1 & \frac{m_1}{M} \\ -\frac{m_1}{M} & -1 \end{pmatrix}, \quad (\text{A2})$$

$$\begin{pmatrix} b_R \\ B_R \end{pmatrix} = V_{bR} \begin{pmatrix} b_R^{(1)} \\ b_R^{(2)} \end{pmatrix}, \quad V_{bR} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad (\text{A3})$$

$$\begin{pmatrix} t_L \\ T_L \end{pmatrix} = V_{tL} \begin{pmatrix} t_L^{(0)} \\ t_L^{(1)} \end{pmatrix}, \quad V_{tL} \simeq \begin{pmatrix} -1 & \frac{m_1}{M} \\ -\frac{m_1}{M} & -1 \end{pmatrix}, \quad (\text{A4})$$

and

$$\begin{pmatrix} t_R \\ T_R \end{pmatrix} = V_{tR} \begin{pmatrix} t_R^{(1)} \\ t_R^{(2)} \end{pmatrix}, \quad V_{tR} \simeq \begin{pmatrix} -\frac{m'_t}{M} & 1 \\ -1 & -\frac{m'_t}{M} \end{pmatrix}. \quad (\text{A5})$$

Note that we approximate the b quark as massless, which will be sufficient for the computation of the one-loop corrections to the vertex and wavefunction-mixing diagrams. Assuming $M \gg m_1, m'_t$, the t quark mass is evaluated as

$$m_t \simeq \frac{m_1 m'_t}{M}. \quad (\text{A6})$$

The heavy KK quarks T and B are almost degenerate

$$m_T \simeq m_B \simeq M. \quad (\text{A7})$$

We next turn to the Nambu-Goldstone boson sector. We define π_W and $\pi_{W'}$ as

$$\begin{pmatrix} \pi_W \\ \pi_{W'} \end{pmatrix} = V_\pi \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix}, \quad V_\pi = \frac{1}{\sqrt{f_1^2 + f_2^2}} \begin{pmatrix} f_2 & f_1 \\ -f_1 & f_2 \end{pmatrix}. \quad (\text{A8})$$

The $\pi_{W'}$ bosons are eaten by the gauge boson W'^μ , while the π_W bosons are physical and remain massless. As appropriate, we will denote the neutral Nambu-Goldstone boson, which will be eaten by the Z when $g_{W,Y} \neq 0$, by π_Z .

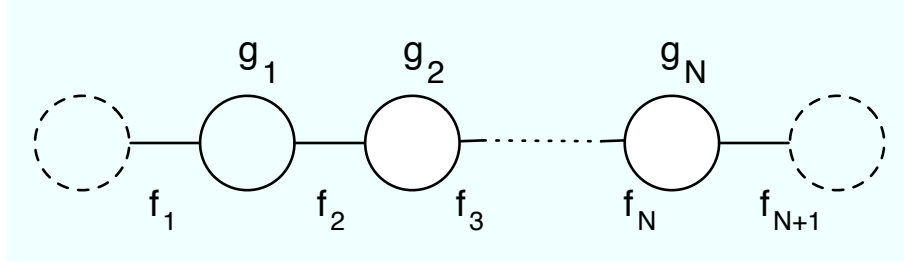


FIG. 6: The N -site global moose. The global-symmetries are at sites 0 and $N + 1$. Left-handed fermions, labeled ψ_{Lj} for $j = 0, 1, \dots, N$, are understood to be at sites 0 to N , and right-handed ones (labeled correspondingly) from sites 1 to $N + 1$.

We are now ready to evaluate various coupling strengths that are relevant for the one-loop computation of the $Zb\bar{b}$ vertex in the gaugeless limit,

$$g_{\bar{b}_L t_R \pi_W} \simeq \sqrt{2} \frac{\sqrt{f_1^2 + f_2^2}}{f_1 f_2} \frac{m_1 m'_t}{M} \simeq \sqrt{2} \frac{m_t}{v}, \quad (\text{A9})$$

$$g_{\bar{b}_L T_R \pi_W} \simeq \sqrt{2} \frac{m_1 f_2}{f_1 \sqrt{f_1^2 + f_2^2}}, \quad (\text{A10})$$

$$g_{\bar{b}_L t_R \pi_{W'}} \simeq 0, \quad (\text{A11})$$

$$g_{\bar{b}_L T_R \pi_{W'}} \simeq -\frac{\sqrt{2} m_1}{\sqrt{f_1^2 + f_2^2}}, \quad (\text{A12})$$

$$g_{\bar{b}_L B_R \pi_Z} \simeq -\frac{m_1 f_2}{f_1 \sqrt{f_1^2 + f_2^2}}, \quad (\text{A13})$$

$$g_{\bar{B}_L t_R \pi_W} \simeq \sqrt{2} \frac{m'_t f_1}{f_2 \sqrt{f_1^2 + f_2^2}}, \quad (\text{A14})$$

$$g_{\bar{B}_L T_R \pi_W} \simeq \frac{\sqrt{2} M}{\sqrt{f_1^2 + f_2^2}} \left[\frac{f_2}{f_1} \frac{m_1^2}{M^2} + \frac{f_1}{f_2} \frac{m'_t{}^2}{M^2} \right], \quad (\text{A15})$$

$$g_{\bar{B}_L t_R \pi_{W'}} \simeq -\frac{\sqrt{2} m'_t}{\sqrt{f_1^2 + f_2^2}}, \quad (\text{A16})$$

$$g_{\bar{B}_L T_R \pi_{W'}} \simeq \frac{\sqrt{2} M}{\sqrt{f_1^2 + f_2^2}} \left[-\frac{m_1^2}{M^2} + \frac{m'_t{}^2}{M^2} \right], \quad (\text{A17})$$

$$g_{\bar{t}_L t_R \pi_Z} \simeq \frac{\sqrt{f_1^2 + f_2^2}}{f_1 f_2} \frac{m_1 m'_t}{M} \simeq \frac{m_t}{v}, \quad (\text{A18})$$

$$g_{\bar{T}_L t_R \pi_Z} \simeq -\frac{m'_t f_1}{f_2 \sqrt{f_1^2 + f_2^2}}, \quad (\text{A19})$$

$$g_{\bar{T}_L T_R \pi_Z} \simeq \frac{M}{\sqrt{f_1^2 + f_2^2}} \left[\frac{f_1}{f_2} \frac{m'_t{}^2}{M^2} + \frac{f_2}{f_1} \frac{m_1^2}{M^2} \right]. \quad (\text{A20})$$

In these expressions we have ignored terms of $\mathcal{O}(m_1/M)^2$ and $\mathcal{O}(m'_t/M)^2$; note that the couplings in eqns. (A15), (A17), and (A20) are enhanced by the (potentially large) factor $(M/\sqrt{f_1^2 + f_2^2})$.

APPENDIX B: CHIRAL CURRENTS IN THE N -SITE MODEL

The generalization of the calculation of the chiral currents of eqn. (25) to the N -site global moose, illustrated in Fig. 6, is straightforward. In this case, the transformation properties of the nonlinear sigma-model fields are

$$\Sigma_1 \rightarrow L \Sigma_1 H_1^\dagger, \quad (B1)$$

$$\Sigma_i \rightarrow H_{i-1} \Sigma_i H_i^\dagger, \quad i = 1, \dots, N \quad (B2)$$

$$\Sigma_{N+1} \rightarrow H_N \Sigma_{N+1} R. \quad (B3)$$

Defining $\Sigma_j = \exp(2i\pi_j/f_j)$, the corresponding infinitesimal transformations are

$$\frac{2\pi_1}{f_1} \rightarrow \frac{2\pi_1}{f_1} + \alpha_L - h_1, \quad (B4)$$

$$\frac{2\pi_i}{f_i} \rightarrow \frac{2\pi_i}{f_i} + h_{i-1} - h_i, \quad i = 1, \dots, N \quad (B5)$$

$$\frac{2\pi_{N+1}}{f_{N+1}} \rightarrow \frac{2\pi_{N+1}}{f_{N+1}} + h_N - \alpha_R, \quad (B6)$$

$$(B7)$$

for infinitesimal $SU(2)_{L,R}$ transformations parameterized by $\alpha_{L,R}$. Unitary gauge corresponds to imposing the condition,

$$f_{i-1}\pi_{i-1} = f_i\pi_i, \quad i = 1, \dots, N+1. \quad (B8)$$

In solving these equations for $\alpha_{L,R} \neq 0$, it is convenient to define

$$\frac{1}{F^2} = \sum_{j=1}^{N+1} \frac{1}{f_j^2}, \quad (B9)$$

$$\frac{1}{F_\ell^2} = \sum_{j=1}^{\ell} \frac{1}{f_j^2}, \quad (B10)$$

$$\frac{1}{\tilde{F}_\ell^2} = \sum_{j=\ell+1}^{N+1} \frac{1}{f_j^2}. \quad (B11)$$

We then find that the transformations imply the fermionic currents

$$j_L^{a\mu} = \bar{\psi}_{L0} \frac{\sigma^a}{2} \gamma^\mu \psi_{L0} + \sum_{k=1}^N a_k \left[\bar{\psi}_{Lk} \frac{\sigma^a}{2} \gamma^\mu \psi_{Lk} + \bar{\psi}_{Rk} \frac{\sigma^a}{2} \gamma^\mu \psi_{Rk} \right], \quad (B12)$$

$$j_R^{a\mu} = \bar{\psi}_{N+1k} \frac{\sigma^a}{2} \gamma^\mu \psi_{N+1k} + \sum_{k=1}^N b_k \left[\bar{\psi}_{Lk} \frac{\sigma^a}{2} \gamma^\mu \psi_{Lk} + \bar{\psi}_{Rk} \frac{\sigma^a}{2} \gamma^\mu \psi_{Rk} \right], \quad (B13)$$

with

$$a_k = \frac{F^2}{\tilde{F}_k^2}, \quad (B14)$$

$$b_k = \frac{F^2}{F_k^2}. \quad (B15)$$

Note that $a_k + b_k \equiv 1$, and therefore the vector currents $j_V^{a\mu} = j_L^{a\mu} + j_R^{a\mu}$ are of canonical form.

These results could be used in more complicated Higgsless models, and suggest that the kinds of corrections which we have found in the 3-site model occur more generally in models with fermion delocalization.

APPENDIX C: WARD-TAKAHASHI IDENTITIES

In this appendix we review the Ward-Takahashi identity that forms the basis of the “gaugeless” limit used in our computation of the one-loop correction to the $Z \rightarrow b\bar{b}$ amplitude. We begin by reviewing the standard model result [21, 22], and then present the generalization to the three-site model.

1. Standard Model [21, 22]

In the gaugeless limit of the standard model (SM), the Z -boson (as a classical field) couples to the conserved current

$$J^\mu = \hat{J}^\mu - M_Z \partial^\mu \pi_Z, \quad (C1)$$

where \hat{J}^μ is the fermionic contribution to the current

$$\hat{J}^\mu = g_{Zbb}^L \bar{b}_L \gamma^\mu P_L b_L + g_{Zbb}^R \bar{b}_R \gamma^\mu P_R b_R + \dots \quad (C2)$$

The Ward-Takahashi identity arising from the current conservation is

$$\begin{aligned} \partial_\mu^x \langle 0 | T \hat{J}^\mu(x) b(y) \bar{b}(z) | 0 \rangle &= M_Z \langle 0 | T (\square_x \pi_Z(x)) b(y) \bar{b}(z) | 0 \rangle - \delta(x-y) (g_{Zbb}^L P_L + g_{Zbb}^R P_R) \langle 0 | b(x) \bar{b}(z) | 0 \rangle \\ &\quad + \delta(x-z) \langle 0 | b(y) \bar{b}(x) | 0 \rangle (g_{Zbb}^L P_R + g_{Zbb}^R P_L). \end{aligned} \quad (C3)$$

In momentum space, we have the relationship between the (connected, but not 1PI) Green's functions

$$\begin{aligned} i(p_1 + p_2)_\mu \langle 0 | \hat{J}^\mu(p_1 + p_2) b(p_2) \bar{b}(p_1) | 0 \rangle &= -M_Z (p_1 + p_2)^2 \langle 0 | \pi_Z(p_1 + p_2) b(p_2) \bar{b}(p_1) | 0 \rangle \\ &\quad - (g_{Zbb}^L P_L + g_{Zbb}^R P_R) S_{bb}(-p_2) + S_{bb}(p_1) (g_{Zbb}^L P_R + g_{Zbb}^R P_L), \end{aligned} \quad (C4)$$

where $S_{bb}(p)$ is the b -quark propagator. We can also write eqn. (C4) in terms of the 1PI Green's functions

$$\begin{aligned} i(p_1 + p_2)_\mu \langle 0 | \hat{J}^\mu(p_1 + p_2) b(p_2) \bar{b}(p_1) | 0 \rangle_{1PI} &= -i M_Z \langle 0 | \pi_Z(p_1 + p_2) b(p_2) \bar{b}(p_1) | 0 \rangle_{1PI} \\ &\quad - S_{bb}^{-1}(p_1) (g_{Zbb}^L P_L + g_{Zbb}^R P_R) + (g_{Zbb}^L P_R + g_{Zbb}^R P_L) S_{bb}^{-1}(-p_2). \end{aligned} \quad (C5)$$

We decompose the amputated Green's functions according to their Lorentz structures¹³ as

$$\begin{aligned} S_{bb}^{-1}(p) &= -i (\not{p} A_{bb}(p^2) - B_{bb}(p^2)) \\ &\simeq -i (\not{p} A_{bb}(m_b^2) - B_{bb}(m_b^2) + \mathcal{O}(p^2 - m_b^2)), \end{aligned} \quad (C6)$$

$$\langle 0 | \hat{J}^\mu(p_1 + p_2) b(p_2) \bar{b}(p_1) | 0 \rangle_{1PI} = \gamma^\mu \left(\langle 0 | \hat{J}^\mu b \bar{b} | 0 \rangle_{\gamma^\mu} \right), \quad (C7)$$

$$\langle 0 | \pi_Z(p_1 + p_2) b(p_2) \bar{b}(p_1) | 0 \rangle_{1PI} = (\not{p}_1 + \not{p}_2) \left(\langle 0 | \pi_Z b \bar{b} | 0 \rangle_{\not{p}} \right) + \gamma_5 \left(\langle 0 | \pi_Z b \bar{b} | 0 \rangle_{\gamma_5} \right), \quad (C8)$$

and where the components of the propagator can be decomposed into two chirality components

$$\begin{aligned} A_{bb}(p^2) &= A_{bb}^L(p^2) P_L + A_{bb}^R(p^2) P_R, \\ B_{bb}(p^2) &= B_{bb}^L(p^2) P_L + B_{bb}^R(p^2) P_R. \end{aligned} \quad (C9)$$

The Ward-Takahashi identity in eqn. (C5) then gives us the conditions [21, 22]

$$i \langle 0 | \hat{J}^\mu b \bar{b} | 0 \rangle_{\gamma^\mu} = -i M_Z \langle 0 | \pi_Z b \bar{b} | 0 \rangle_{\not{p}} + i A_{bb}(m_b^2) (g_{Zbb}^L P_L + g_{Zbb}^R P_R), \quad (C10)$$

$$0 = -M_Z \gamma_5 \langle 0 | \pi_Z b \bar{b} | 0 \rangle_{\gamma_5} - B_{bb}(m_b^2) (g_{Zbb}^L - g_{Zbb}^R) (P_L - P_R), \quad (C11)$$

where we can project out different chiral structures in eqn. (C10).

At tree level in the standard model we have,

$$\langle 0 | \pi_Z b \bar{b} | 0 \rangle_{\not{p}}^{\text{tree}} = 0, \quad (C12)$$

$$A_{bb}(0)^{\text{tree}} = 1, \quad (C13)$$

$$B_{bb}(0)^{\text{tree}} = m_b \quad (C14)$$

so eqn. (C10) gives us the tree-level $Zb\bar{b}$ coupling, and eqn. (C11) relates the tree-level $\pi_Z b\bar{b}$ coupling to the mass of the bottom quark.

¹³ In eqn. (C8) we note that the amplitude has, in general, both scalar and pseudoscalar parts. In the conventional bases, the fermion masses are real and the scalar part vanishes.

To compute the $Z \rightarrow \bar{b}_L b_L$ amplitude at one-loop order, we must multiply eqn. (C10) by the wavefunction renormalization of the bottom quark according to the LSZ reduction formula and project out the left-handed chirality component to find

$$\begin{aligned} i g_{Z\bar{b}_L b_L}^{1\text{-loop}} P_L &= i \sqrt{Z_b} \langle 0 | \hat{J}^\mu b \bar{b} | 0 \rangle |_{\gamma^\mu} \sqrt{Z_b} P_L \\ &= -i M_Z \sqrt{Z_b} \langle 0 | \pi_Z b \bar{b} | 0 \rangle |_{\not{p}} \sqrt{Z_b} P_L + i \sqrt{Z_b} A_{bb} (m_b^2) (g_{Zbb}^L P_L + g_{Zbb}^R P_R) \sqrt{Z_b} P_L. \end{aligned} \quad (C15)$$

At one-loop order we may write the wavefunction renormalization as

$$\begin{aligned} \sqrt{Z_b} &= \sqrt{Z_{b_L}} P_L + \sqrt{Z_{b_R}} P_R \\ &= 1 + \frac{1}{2} \delta Z_{b_L} P_L + \frac{1}{2} \delta Z_{b_R} P_R \end{aligned} \quad (C16)$$

where the $\delta Z_{b_L, b_R}$ are of one-loop order. We also note that, to one-loop order, we have

$$A_{bb}(p^2) = 1 - \delta Z_{b_L} P_L - \delta Z_{b_R} P_R. \quad (C17)$$

The $Z b_L b_L$ coupling at one-loop order is then,

$$i g_{Z\bar{b}_L b_L}^{1\text{-loop}} = i g_{Zbb} - i M_Z \langle 0 | \pi_Z b \bar{b} | 0 \rangle |_{\not{p}}. \quad (C18)$$

Note that the corrections due to the wavefunction renormalization have canceled, and we have only to calculate the $\pi_Z \bar{b}_L b_L$ coupling to one-loop [21, 22].

2. Ward-Takahashi Identity in the Three-Site Higgsless Model

In the gaugeless limit of the three-site model, the Z -boson couples to the conserved current

$$J_{3\text{-site}}^\mu = \hat{J}_{3\text{-site}}^\mu - M_Z \partial^\mu \pi_Z, \quad (C19)$$

with fermionic contributions which now involve both “diagonal” and “off-diagonal” terms

$$\hat{J}_{3\text{-site}}^\mu = \bar{b} \gamma^\mu g_{Zbb} b + (\bar{B} \gamma^\mu g_{ZBb} b + \text{h.c.}) + \dots, \quad (C20)$$

where, for convenience, we define

$$g_{Zbb} \equiv g_{Zbb}^L P_L + g_{Zbb}^R P_R, \quad (C21)$$

and also the coupling

$$\tilde{g}_{Zbb} \equiv g_{Zbb}^L P_R + g_{Zbb}^R P_L, \quad (C22)$$

and similarly define g_{ZBb} and \tilde{g}_{ZBb} . The Ward-Takahashi identity arising from the current conservation is

$$\begin{aligned} \partial_\mu^x \langle 0 | T \hat{J}_{3\text{-site}}^\mu(x) b(y) \bar{b}(z) | 0 \rangle &= M_Z \langle 0 | T (\Box_x \pi_Z(x)) b(y) \bar{b}(z) | 0 \rangle - \delta(x-y) [g_{Zbb} \langle 0 | b(x) \bar{b}(z) | 0 \rangle + g_{ZBb} \langle 0 | B(x) \bar{b}(z) | 0 \rangle] \\ &\quad + \delta(x-z) [\langle 0 | b(y) \bar{b}(x) | 0 \rangle \tilde{g}_{Zbb} + \langle 0 | b(y) \bar{B}(x) | 0 \rangle \tilde{g}_{ZBb}]. \end{aligned} \quad (C23)$$

In momentum space, we have the relationship between the (connected) Green’s functions

$$\begin{aligned} i(p_1 + p_2)_\mu \langle 0 | \hat{J}_{3\text{-site}}^\mu(p_1 + p_2) b(p_2) \bar{b}(p_1) | 0 \rangle &= -M_Z (p_1 + p_2)^2 \langle 0 | \pi_Z(p_1 + p_2) b(p_2) \bar{b}(p_1) | 0 \rangle \\ &\quad - g_{Zbb} S_{bb}(-p_2) - g_{ZBb} S_{Bb}(-p_2) + S_{bb}(p_1) \tilde{g}_{Zbb} + S_{bB}(p_1) \tilde{g}_{ZBb}. \end{aligned} \quad (C24)$$

Note that, compared to the SM (see eqn. (C4)), eqn. (C24) contains additional terms S_{Bb} and S_{bB} because \hat{J}_μ contains $\bar{B} \gamma^\mu b$ and $\bar{b} \gamma^\mu B$ contributions. Also, as in the case of the SM, the Green’s functions and couplings are four-component matrices in Dirac spinor space which can be separated in terms of their chiral structure.

In the presence of $B - b$ mixing, we have to take into account that the connected Green’s functions can involve non-1PI fermion-mixing diagrams. For example,

$$\begin{aligned} \langle 0 | \hat{J}_{3\text{-site}}^\mu(p_1 + p_2) b(p_2) \bar{b}(p_1) | 0 \rangle &= S_{bb} \langle 0 | \hat{J}_{3\text{-site}}^\mu b \bar{b} | 0 \rangle_{1\text{PI}} S_{bb} + S_{bB} \langle 0 | \hat{J}_{3\text{-site}}^\mu B \bar{b} | 0 \rangle_{1\text{PI}} S_{bb} \\ &\quad + S_{bb} \langle 0 | \hat{J}_{3\text{-site}}^\mu b \bar{B} | 0 \rangle_{1\text{PI}} S_{Bb} + S_{bB} \langle 0 | \hat{J}_{3\text{-site}}^\mu B \bar{B} | 0 \rangle_{1\text{PI}} S_{Bb}, \end{aligned} \quad (C25)$$

where $\langle \rangle_{1\text{PI}}$, as before, are the 1PI Green's functions, and where $S_{bB}(p)$ is the “off-diagonal” fermion propagator. The Ward-Takahashi identity involving the 1PI Green's functions is then

$$\begin{aligned}
& \mathbf{i}(p_1 + p_2)_\mu \left\{ \langle 0 | \hat{J}_{3\text{-site}}^\mu(p_1 + p_2) b(p_2) \bar{b}(p_1) | 0 \rangle_{1\text{PI}} \right. \\
& \quad + S_{bb}^{-1}(p_1) S_{bB}(p_1) \langle 0 | \hat{J}_{3\text{-site}}^\mu(p_1 + p_2) b(p_2) \bar{B}(p_1) | 0 \rangle_{1\text{PI}} \\
& \quad + \langle 0 | \hat{J}_{3\text{-site}}(p_1 + p_2) B(p_2) \bar{b}(p_1) | 0 \rangle_{1\text{PI}} S_{Bb}(-p_2) S_{bb}^{-1}(-p_2) \\
& \quad \left. + S_{bb}^{-1}(p_1) S_{bB}(p_1) \langle 0 | \hat{J}_{3\text{-site}}^\mu(p_1 + p_2) B(p_2) \bar{B}(p_1) | 0 \rangle_{1\text{PI}} S_{Bb}(-p_2) S_{bb}^{-1}(-p_2) \right\} \\
& = -\mathbf{i} M_Z \left\{ \langle 0 | \pi_Z(p_1 + p_2) b(p_2) \bar{b}(p_1) | 0 \rangle_{1\text{PI}} + S_{bb}^{-1}(p_1) S_{bB}(p_1) \langle 0 | \pi_Z(p_1 + p_2) b(p_2) \bar{B}(p_1) | 0 \rangle_{1\text{PI}} \right. \\
& \quad + \langle 0 | \pi_Z(p_1 + p_2) B(p_2) \bar{b}(p_1) | 0 \rangle_{1\text{PI}} S_{Bb}(-p_2) S_{bb}^{-1} S_{Bb}(-p_2) \\
& \quad + S_{bb}^{-1}(p_1) S_{bB}(p_1) \langle 0 | \pi_Z(p_1 + p_2) B(p_2) \bar{B}(p_1) | 0 \rangle_{1\text{PI}} S_{Bb}(-p_2) S_{bb}^{-1}(-p_2) \Big\} \\
& \quad - S_{bb}^{-1}(p_1) g_{Zbb} - S_{bb}^{-1}(p_1) g_{ZBb} S_{Bb}(-p_2) S_{bb}^{-1}(-p_2) \\
& \quad + \tilde{g}_{Zbb} S_{bb}^{-1}(-p_2) + S_{bb}^{-1}(p_1) S_{bB}(p_1) \tilde{g}_{ZBb} S_{bb}^{-1}(-p_2) . \tag{C26}
\end{aligned}$$

We now work to one-loop order with $p_{1,2}^2 \sim m_b^2$. At this order, we have

$$S_{bb}^{-1}(p) S_{bB}(p) = (-\mathbf{i} \Sigma_{bB}(p)) S_{BB}^{\text{tree}}(p) = \Sigma_{bB}(p) \frac{1}{\not{p} - M_B}, \tag{C27}$$

$$S_{Bb}(p) S_{bb}^{-1}(p) = S_{BB}^{\text{tree}}(p) (-\mathbf{i} \Sigma_{Bb}(p)) = \frac{1}{\not{p} - M_B} \Sigma_{Bb}(p), \tag{C28}$$

where Σ_{bB} is the fermion-mixing self-energy function and $S_{BB}(p)$ is the heavy B -quark propagator. Let

$$\Sigma_{bB}(p) = -\not{p} \delta Z_{bB} + \delta M_{bB}, \tag{C29}$$

where, again, δZ_{bB} and δM_{bB} are matrices in four-component Dirac spinor space. The left-hand side of eqn. (C26) simplifies to

$$\begin{aligned}
& (\text{LHS of eqn. C26}) \\
& = \mathbf{i}(p_1 + p_2)_\mu \left\{ \langle 0 | \hat{J}_{3\text{-site}}^\mu b \bar{b} | 0 \rangle_{1\text{PI}} + \Sigma_{bB}(p_1) \frac{1}{\not{p}_1 - M_B} \gamma^\mu g_{ZBb} + \gamma^\mu g_{ZBb} \frac{1}{-\not{p}_2 - M_B} \Sigma_{Bb}(-p_2) \right\}. \tag{C30}
\end{aligned}$$

The right-hand side of eqn. (C26) simplifies to

$$\begin{aligned}
& (\text{RHS of eqn. C26}) = -M_Z \langle 0 | \pi_Z b \bar{b} | 0 \rangle_{1\text{PI}} - \mathbf{i} S_{bb}^{-1}(p_1) g_{Zbb} + \tilde{g}_{Zbb} S_{bb}^{-1}(-p_2) \\
& \quad + \Sigma_{bB}(p_1) \frac{1}{\not{p}_1 - M_B} \left\{ \mathbf{i} M_Z g_{\pi_Z \bar{B}b} - \tilde{g}_{ZBb} S_{bb}^{-1}(-p_2) \right\} \\
& \quad + \left\{ \mathbf{i} M_Z g_{\pi_Z \bar{b}B} + S_{bb}^{-1}(p_1) g_{ZBb} \right\} \frac{1}{-\not{p}_2 - M_B} \Sigma_{Bb}(-p_2). \tag{C31}
\end{aligned}$$

Using the tree-level relations in the limit of vanishing m_b

$$M_Z g_{\pi_Z \bar{b}B} = -\mathbf{i} M_B \tilde{g}_{ZBb}, \tag{C32}$$

$$M_Z g_{\pi_Z \bar{B}b} = \mathbf{i} M_B g_{ZBb}, \tag{C33}$$

$$S_{bb}^{-1}(p) = -\mathbf{i} \not{p}, \tag{C34}$$

we can further simplify eqn. (C31)

$$\begin{aligned}
& (\text{RHS of eqn. C26}) \\
& = -M_Z \langle 0 | \pi_Z b \bar{b} | 0 \rangle_{1\text{PI}} - S_{bb}^{-1}(p_1) g_{Zbb} + \tilde{g}_{Zbb} S_{bb}^{-1}(-p_2) \\
& \quad + \mathbf{i} \Sigma_{bB}(p_1) \frac{1}{\not{p}_1 - M_B} (\not{p}_1 + \not{p}_2) g_{ZBb} + \mathbf{i} (\not{p}_1 + \not{p}_2) g_{ZBb} \frac{1}{-\not{p}_2 - M_B} \Sigma_{Bb}(-p_2) \\
& \quad - \frac{M_Z}{M_B} \Sigma_{bB}(p_1) g_{\pi_Z \bar{B}b} - \frac{M_Z}{M_B} g_{\pi_Z \bar{b}B} \Sigma_{Bb}(-p_2). \tag{C35}
\end{aligned}$$

Combining Eqs. (C30) and (C35), we have three-site Ward-Takahashi identity at one-loop (for $m_b = 0$)

$$\begin{aligned} i(p_1 + p_2)_\mu \langle 0 | \hat{j}_{3\text{-site}}^\mu b \bar{b} | 0 \rangle_{1\text{PI}} = & -M_Z \left\{ \langle 0 | \pi_Z b \bar{b} | 0 \rangle_{1\text{PI}} + \frac{\Sigma_{bB}(p_1)}{M_B} g_{\pi_Z \bar{B}b} + g_{\pi_Z \bar{b}B} \frac{\Sigma_{Bb}(-p_2)}{M_B} \right\} \\ & - S_{bb}^{-1}(p_1) g_{Zbb} + \tilde{g}_{Zbb} S_{bb}^{-1}(-p_2). \end{aligned} \quad (\text{C36})$$

Compared to the Ward-Takahashi Identity in the standard model (eqn. (C5)), we have the additional terms due to the $B - b$ mixing. At tree-level, however, these effects vanish and we simply have the standard model results.

To compute the $Z \bar{b}_L b_L$ amplitude at one-loop, we have to separate the different contributions of the amplitude according to their Lorentz structure, and collect terms proportional to $(\not{p}_1 + \not{p}_2)$. Since the leading term in the top line as well as the terms in the bottom line of eqn. (C36) are present in the standard model Ward-Takahashi identity, their contribution to the $Z \bar{b}_L b_L$ amplitude is presented in eqn. (C18). (There are, however, additional diagrams in the three-site model that contribute to $\langle 0 | \pi_Z b \bar{b} | 0 \rangle$.) As for the contributions due to $B - b$ mixing, it is only the kinetic mixing contributions of Σ_{bB} that contribute to $Z \bar{b}_L b_L$ because we are interested only in terms proportional to \not{p} in Σ_{bB} . We note that

$$\left\{ \Sigma_{Bb}(p) P_L \right\} \Big|_{\not{p}} = \left\{ \Sigma_{bB}(p) P_L \right\} \Big|_{\not{p}} = -\delta Z_{b_L B_L}, \quad (\text{C37})$$

$$g_{\pi_Z \bar{B}b} = -\tilde{g}_{\pi_Z \bar{b}B}, \quad (\text{C38})$$

so the $Z b_L b_L$ coupling of the three-site model at one-loop order is

$$i g_{Z \bar{b}_L b_L}^{3\text{-Site}} = i g_{Z \bar{b}_L b_L} - i M_Z \left\{ \langle 0 | \pi_Z b \bar{b} | 0 \rangle \Big|_{\not{p}} - g_{\pi_Z \bar{B}b_L} \frac{\delta Z_{b_L B_L}}{M_B} \right\}. \quad (\text{C39})$$

In terms of Feynman diagrams, this is the same as the calculation performed in the body of the paper.

APPENDIX D: UNITARY GAUGE CALCULATION

In this appendix we briefly describe the computation of $Z \rightarrow b \bar{b}$ in unitary gauge, without recourse to the gaugeless limit. In order to do so we must consider the effects of mass and wavefunction mixing in the bottom sector. At the one-loop level, the tree-level mass eigenstate b_L mixes with both B_L and B_R , and these mixings give contributions (in addition to those from the triangle diagrams) to the tree-level Z -boson couplings

$$\mathcal{L}_Z = g_{Zbb}^L Z_\mu \bar{b}_L \gamma^\mu b_L + g_{ZBb}^L Z_\mu (\bar{b}_L \gamma^\mu B_L + \bar{B}_L \gamma^\mu b_L). \quad (\text{D1})$$

Let us parameterize the mass and wavefunction mixing through the Lagrangian¹⁴

$$\begin{aligned} \mathcal{L} = & i \begin{pmatrix} \bar{b}_L \\ \bar{B}_L \end{pmatrix}^T \not{\partial} \begin{pmatrix} 1 + \delta Z_{b_L} & \delta Z_{B_L b_L} \\ \delta Z_{B_L b_L} & 1 \end{pmatrix} \begin{pmatrix} b_L \\ B_L \end{pmatrix} + i \begin{pmatrix} \bar{b}_R \\ \bar{B}_R \end{pmatrix}^T \not{\partial} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} b_R \\ B_R \end{pmatrix} \\ & - \left[(\bar{b}_L \quad \bar{B}_L) \begin{pmatrix} m_b + \delta m_b & \delta M_{bB} \\ 0 & M \end{pmatrix} \begin{pmatrix} b_R \\ B_R \end{pmatrix} + \text{h.c.} \right]. \end{aligned} \quad (\text{D2})$$

The one-loop canonical mass eigenstates (denoted with a superscript r) are related to the tree-level eigenstates by

$$\begin{pmatrix} b_L \\ B_L \end{pmatrix} = \begin{pmatrix} 1 - \frac{\delta Z_{b_L}}{2} & \frac{\delta M_{bB}}{M} - \delta Z_{B_L b_L} \\ -\frac{\delta M_{bB}}{M} & 1 \end{pmatrix} \begin{pmatrix} b_L^r \\ B_L^r \end{pmatrix}, \quad (\text{D3})$$

$$\begin{pmatrix} b_R \\ B_R \end{pmatrix} = \begin{pmatrix} 1 & \frac{m_b}{M} \left(\frac{\delta M_{bB}}{M} - \delta Z_{B_L b_L} \right) \\ -\frac{m_b}{M} \left(\frac{\delta M_{bB}}{M} - \delta Z_{B_L b_L} \right) & 1 \end{pmatrix} \begin{pmatrix} b_R^r \\ B_R^r \end{pmatrix}. \quad (\text{D4})$$

In terms of the one-loop mass eigenstates (eqns. (D3) and (D4)), we find

$$\mathcal{L}_Z = \left[g_{Zbb}^L (1 - \delta Z_{b_L}) - 2 g_{ZBb}^L \frac{\delta M_{bB}}{M} \right] Z_\mu \bar{b}_L^r \gamma^\mu b_L^r. \quad (\text{D5})$$

¹⁴ Here we neglect terms that will not contribute to the $Z \rightarrow b \bar{b}$ process that we are computing. Also, for completeness, we include contributions of order m_b in the right-handed sector; these are not necessary for the computation here, but would be necessary to implement the calculation described in footnote 11.

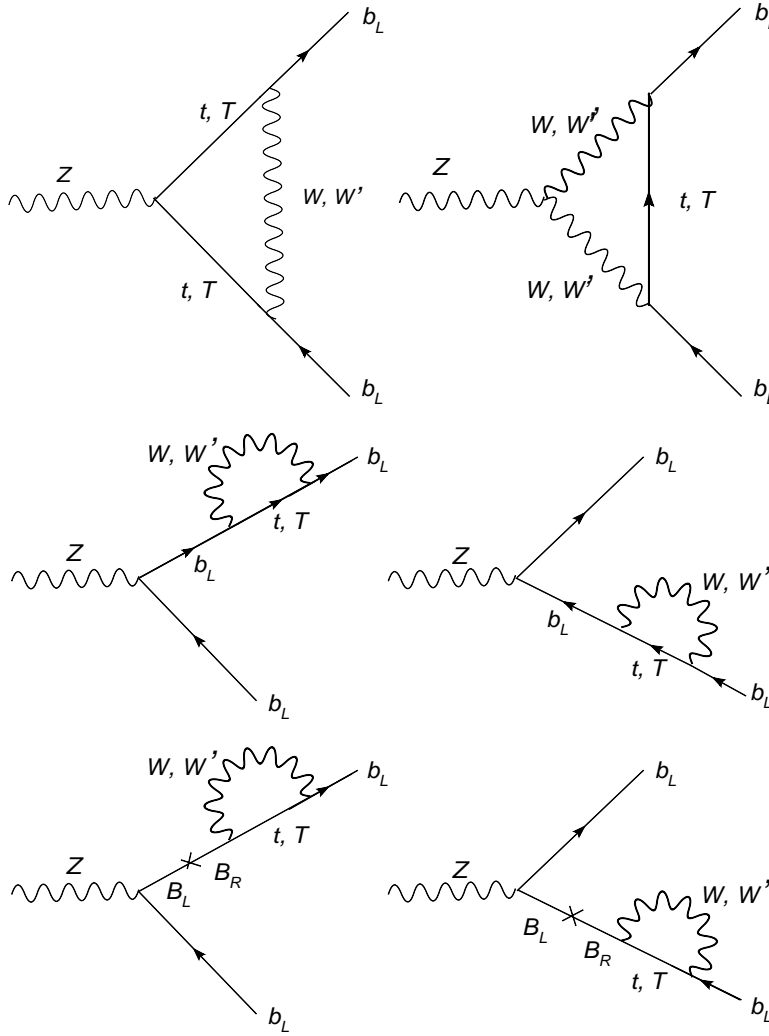


FIG. 7: Diagrams that contribute to δg_{Zbb} in the unitary gauge. The mass and wavefunction mixing diagrams implement the results of eqn. (D5).

Computing the diagrams illustrated in Fig. 7 in unitary gauge, using dimensional regularization and $\overline{\text{MS}}$, and subtracting the corresponding diagrams for the d or s quarks (to isolate the flavor-dependent correction), we reproduce eqn. (40).

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